



**TECHNICAL REPORT RD-MG-96-32**

**A METHOD FOR THE DETERMINATION OF TARGET  
ASPECT ANGLE WITH RESPECT TO A RADAR**

**Willie D. Caraway III  
David C. Abell  
Missile Guidance Directorate  
Research, Development, and Engineering Center**

**April 1996**

*THIS QUALITY INSPECTED 2*



**U.S. ARMY MISSILE COMMAND**

***Redstone Arsenal, Alabama* 35898-5000**

*Approved for public release; distribution is unlimited.*

**19960611 086**

### **DESTRUCTION NOTICE**

**FOR CLASSIFIED DOCUMENTS, FOLLOW THE PROCEDURES IN DoD 5200.22-M, INDUSTRIAL SECURITY MANUAL, SECTION II-19 OR DoD 5200.1-R, INFORMATION SECURITY PROGRAM REGULATION, CHAPTER IX. FOR UNCLASSIFIED, LIMITED DOCUMENTS, DESTROY BY ANY METHOD THAT WILL PREVENT DISCLOSURE OF CONTENTS OR RECONSTRUCTION OF THE DOCUMENT.**

### **DISCLAIMER**

**THE FINDINGS IN THIS REPORT ARE NOT TO BE CONSTRUED AS AN OFFICIAL DEPARTMENT OF THE ARMY POSITION UNLESS SO DESIGNATED BY OTHER AUTHORIZED DOCUMENTS.**

### **TRADE NAMES**

**USE OF TRADE NAMES OR MANUFACTURERS IN THIS REPORT DOES NOT CONSTITUTE AN OFFICIAL ENDORSEMENT OR APPROVAL OF THE USE OF SUCH COMMERCIAL HARDWARE OR SOFTWARE.**

UNCLASSIFIED

## REPORT DOCUMENTATION PAGE

Form Approved  
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>			1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.	
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE				
4. PERFORMING ORGANIZATION REPORT NUMBER(S) <b>TR-RD-MG-96-32</b>			5. MONITORING ORGANIZATION REPORT NUMBER(S)	
6a. NAME OF PERFORMING ORGANIZATION Missile Guidance Directorate RDEC		6b. OFFICE SYMBOL (If Applicable) AMSMI-RD-MG-RF		7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) Command, U. S. Army Missile Command ATTN: AMSMI-RD-MG-RF Redstone Arsenal, AL 35898			7b. ADDRESS (City, State, and ZIP Code)	
8a. NAME OF FUNDING/SPONSORING ORGANIZATION		8b. OFFICE SYMBOL (If Applicable)		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code)			10. SOURCE OF FUNDING NUMBERS	
			PROGRAM ELEMENT NO.	PROJECT NO.
			TASK NO.	WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification) A Method for the Determination of Target Aspect Angle with Respect to a Radar				
12. PERSONAL AUTHOR(S) Caraway, Willie D. III and Abell, David C.				
13a. TYPE OF REPORT Final		13b. TIME COVERED FROM _____ TO _____		14. DATE OF REPORT (Year, Month, Day) April 1996
15. PAGE COUNT 18				
16. SUPPLEMENTARY NOTATION				
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB-GROUP	Aspect Angle, Non-Cooperative Target Recognition (NCTR), Advanced Range Data System (ARDS), Multi-Role Survival Radar (MRSR)	
19. ABSTRACT (Continue on reverse if necessary and identify by block number)				
<p>In the post Gulf War era, significant effort has been expended in the area of Non-Cooperative Target Identification (NCTI). The techniques proposed for use with radar systems all have a common feature: the need to use target aspect angle with respect to (w.r.t.) the direction of arrival of the radar signal to reduce the problem to a manageable size. This paper will present a rigorous development of the mathematics for solving this problem exactly given target position and attitude data; and then, develop the mathematics for solving this problem when only the radar's positional measurement of the target is available. Finally, data will be presented to show the practical accuracy limits of aspect angle calculations using radar data.</p>				
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS			21. ABSTRACT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>	
22a. NAME OF RESPONSIBLE INDIVIDUAL Willie D. Caraway			22b. TELEPHONE (Include Area Code) (205) 876 - 5156	22c. OFFICE SYMBOL AMSMI-RD-MG-RF

## TABLE OF CONTENTS

	<u>Page</u>
I. INTRODUCTION.....	1
II. DEFINITIONS .....	1
III. CLOSED FORM SOLUTION .....	2
A. $\theta$ and $\phi$ w.r.t. the Aircraft's Reference Frame .....	2
B. Development of the Radar to Aircraft Pointing Vector, $\hat{V}_{RA}$ , w.r.t the Radar's Reference Frame.....	3
C. Development of the Aircraft to Radar Pointing Vector, $\hat{V}'_{AR}$ , w.r.t. the Aircraft's Frame of Reference.....	3
IV. PRACTICAL CONSIDERATIONS FOR ASPECT ANGLES CALCULATION .....	8
V. AIRCRAFT TESTS INVOLVING ASPECT ANGLES.....	11

## LIST OF ILLUSTRATIONS

<b><u>Figure</u></b>	<b><u>Title</u></b>	<b><u>Page</u></b>
1.	Aircraft Aspect Angles .....	2
2.	Rotation due to $\xi$ .....	4
3.	Rotation due to $\eta$ .....	5
4.	Rotation due to $\zeta$ .....	6
5.	Simplified Estimate of $\theta$ .....	10
6.	Simplified Estimate of $\phi$ .....	10

## I. INTRODUCTION

In the post Gulf War era, significant effort has been expended in the area of Non-Cooperative Target Identification (NCTI). The techniques proposed for use with radar systems all have a common feature: the need to use target aspect angle with respect to (w.r.t.) the direction of arrival of the radar signal to reduce the problem to a manageable size. This report will present a rigorous development of the mathematics for solving this problem exactly given target position and attitude data; and then, develop the mathematics for solving this problem when only the radar's positional measurement of the target is available. Finally, data will be presented to show the practical accuracy limits of aspect angle calculations using radar data.

## II. DEFINITIONS

The basic problem is to find, w.r.t. the aircraft, the two angles that describe the direction of arrival of a radar beam at the aircraft. Call these angles  $\theta$  and  $\phi$  where  $\theta$  is the angle between the aircraft's nose and the projection of the radar beam into the aircraft's wing plane measured in the counter-clockwise direction when viewed from above and  $\phi$  is the angle between the aircraft's wing plane and the vector that describes the radar beam striking the aircraft measured in the clockwise direction when viewed from the port wing.

The information given to solve this problem is the radar's measurement of the aircraft's current position in a Rng, Az, El coordinate system referenced to the radar. Call this point P and denote it as a function of Rng, Az, and El by  $P(Rng, Az, El)$ . Also given is the aircraft's spacial orientation w.r.t. the radar in the form of a heading angle,  $\xi$ ; a pitch angle,  $\eta$ ; and a roll or bank angle,  $\zeta$ . The heading angle is the angle between true North and the aircraft's centerline in the horizontal plane of the radar measured in the clockwise direction. The pitch angle is the angle between the radar's horizontal plane and the aircraft's center line measured in the clockwise direction. The roll angle is the angle between the radar's horizontal plane and the aircraft's port wing centerline measured in the clockwise direction.

### III. CLOSED FORM SOLUTION

#### A. $\theta$ and $\phi$ w.r.t. the Aircraft's Reference Frame

As a first step in solving this problem, assume a right-handed Cartesian coordinate system with origin at the aircraft and unit vectors  $\hat{x}_a$ ,  $\hat{y}_a$ , and  $\hat{z}_a$  where  $\hat{x}_a$  points in the direction of the aircraft's nose,  $\hat{y}_a$  points out the aircraft's port wing, and  $\hat{z}_a$  points up from the  $x_a y_a$  plane in the right-handed sense (Fig. 1). Let  $\hat{V}_{AR}$  be the pointing vector from the aircraft to the radar and be expressed in the form  $\hat{V}_{AR} = V'_{ARx} \hat{x}_a + V'_{ARy} \hat{y}_a + V'_{ARz} \hat{z}_a$ . Then from Figure 1, the aspect angles  $\theta$  and  $\phi$  are:

$$\theta = \cos^{-1} \left( \frac{V'_{ARx}}{\sqrt{(V'_{ARx})^2 + (V'_{ARy})^2}} \right) \quad (1)$$

$$\phi = \tan^{-1} \left( \frac{V'_{ARz}}{\sqrt{(V'_{ARx})^2 + (V'_{ARy})^2}} \right). \quad (2)$$

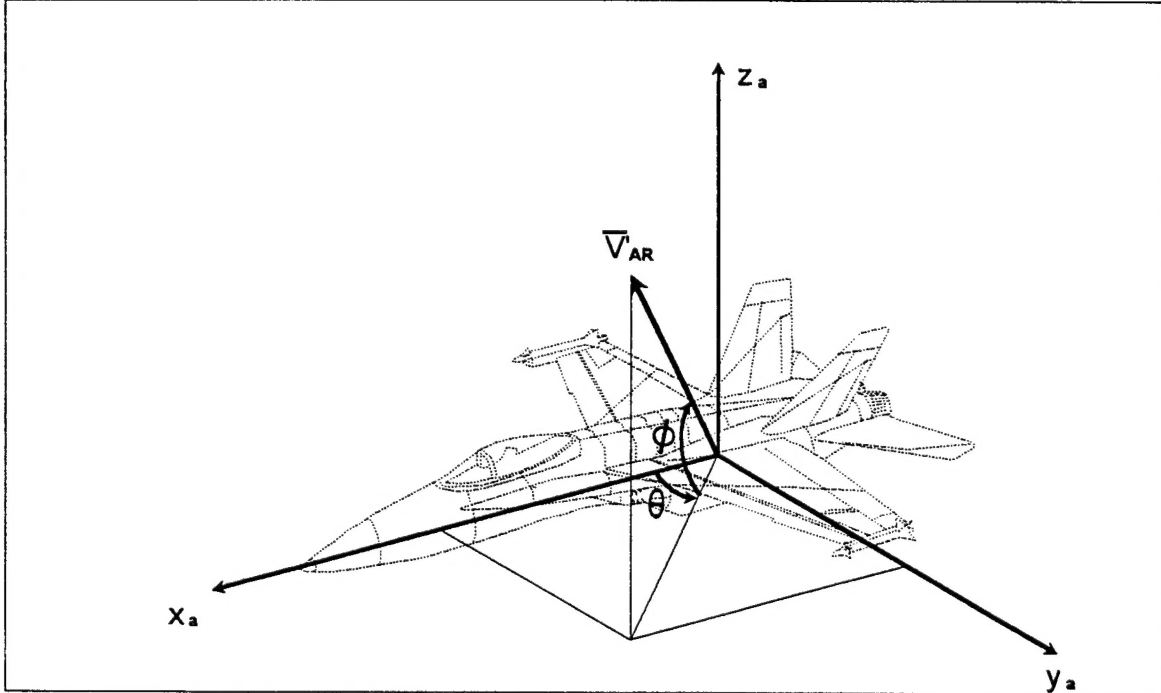


Figure 1. Aircraft Aspect Angles

Thus, the major step in the determination of the direction of arrival of the radar beam w.r.t. the aircraft is the transformation of the radar's measurement of the aircraft's position into a pointing vector from the aircraft to the radar in the coordinate space described above.

### **B. Development of the Radar to Aircraft Pointing Vector, $\hat{V}_{RA}$ , w.r.t the Radar's Reference Frame**

The position data from the radar is typically in a Rng, Az, El coordinate system referenced to the radar. As the Rng, Az, El coordinate system is a left-handed system, three transformations of the radar data point P (Rng, Az, El) will be required to map it into the aircraft's coordinate system.

The first transformation is to map the Rng, Az, El data into a right-handed East, North, Up coordinate system referenced to the radar. This is a standard transformation in the radar field and is given by:

$$\text{East or } x = \text{Rng} \sin(\text{Az}) \cos(\text{El}) \quad (3a)$$

$$\text{North or } y = \text{Rng} \cos(\text{Az}) \cos(\text{El}) \quad (3b)$$

$$\text{Up or } z = \text{Rng} \sin(\text{El}). \quad (3c)$$

From this, a pointing vector from the radar to the aircraft,  $\hat{V}_{RA}$ , can be defined as:

$$\hat{V}_{RA} = \sin(\text{Az})\cos(\text{El})\hat{x} + \cos(\text{Az})\cos(\text{El})\hat{y} + \sin(\text{El})\hat{z}. \quad (4)$$

### **C. Development of the Aircraft to Radar Pointing Vector, $\hat{V}_{AR}$ , w.r.t. the Aircraft's Frame of Reference**

The next transformation is to transform the vector,  $\hat{V}_{RA}$ , into a pointing vector,  $\hat{V}_{AR}$ , from the aircraft to the radar in an East, North, Up coordinate system centered at the aircraft.

Since the aircraft's position is also the head of,  $\bar{V}_{RA}$ ,  $\hat{V}_{AR}$  will simply be the additive inverse of  $\hat{V}_{RA}$  or

$$\hat{V}_{AR} = -1 * \hat{V}_{RA}. \quad (5)$$

The final transformation is to transform the vector  $\hat{V}_{AR}$  into the vector  $\hat{V}'_{AR}$  as defined earlier. While this is not an extremely difficult transformation, it is rather involved since it is effectively a three axis of rotation rigid body problem.



Using vector analysis, this type of transformation can be written as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = J \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \quad (6)$$

where:

$x, y, z$  are the axes labels for the original coordinate system,  
 $x', y', z'$  are the axes labels for the rotated coordinate system,  
and  $J$  is the transformation matrix whose columns consist of the direction cosines of the unit vectors  $\hat{x}', \hat{y}', \hat{z}'$  w.r.t. the old coordinate system.

Now the only remaining piece of the puzzle is the derivation of the transformation matrix  $J$ .

The transformation matrix  $J$  can be derived through three successive transforms since each rotation is a linear operation and therefore separable. The first transform is the result of the heading angle,  $\xi$ , and is a clockwise rotation of the original  $x, y, z$  coordinate system about the  $z$ -axis. This is shown in Figure 2. An adjustment must be made to  $\xi$  though due to a reference difference between the radar and aircraft coordinate systems. The angle  $\xi$  is referenced to North, but the  $x$ -axis of the radar's coordinate system is aligned to East. This results in the need for a new heading angle,  $\xi'$ , where  $\xi' = \xi - 90$  degrees.

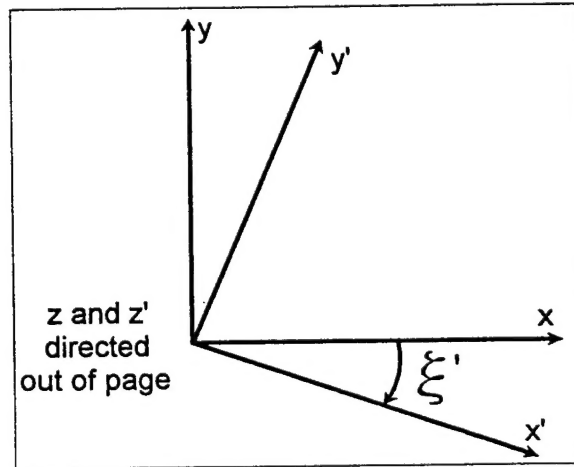


Figure 2. Rotation due to  $\xi$

Using Figure 2, the transformation is:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos(\xi') & \sin(\xi') & 0 \\ -\sin(\xi') & \cos(\xi') & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}. \quad (7)$$

The next transform is the result of the pitch angle,  $\eta$ , and is a clockwise rotation of the  $x', y', z'$  coordinate system about the  $y'$ -axis. This is shown in Figure 3.

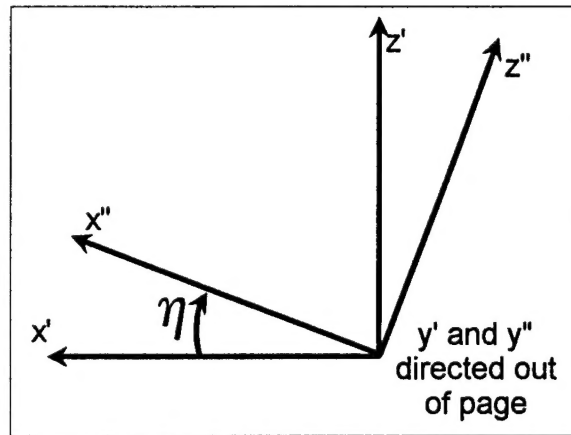


Figure 3. Rotation due to  $\eta$

Using Figure 3, the transformation is:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos(\eta) & 0 & -\sin(\eta) \\ 0 & 1 & 0 \\ \sin(\eta) & 0 & \cos(\eta) \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix}. \quad (8)$$

The final transform is the result of the roll angle,  $\zeta$ , and is a clockwise rotation of the  $x''$ ,  $y''$ ,  $z''$  coordinate system about the  $x''$  axis. This is shown in Figure 4.

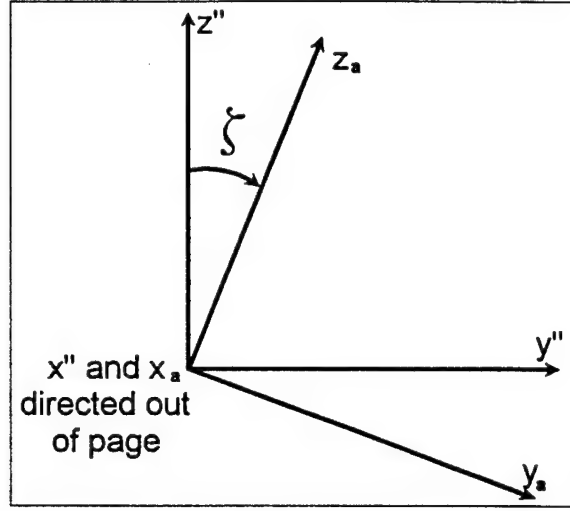


Figure 4. Rotation due to  $\zeta$

Using Figure 4, the transformation is:

$$\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \cos(\zeta) & \sin(\zeta) \\ 1 & -\sin(\zeta) & \cos(\zeta) \end{bmatrix} \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix}. \quad (9)$$

Now substituting Equations (8) and (9) into (7) yields:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos(\xi')\cos(\eta) & \cos(\xi')\sin(\eta)\sin(\zeta) + \sin(\xi')\cos(\zeta) \\ -\sin(\xi')\cos(\eta) & -\sin(\xi')\sin(\eta)\sin(\zeta) + \cos(\xi')\cos(\zeta) \\ \sin(\eta) & -\cos(\eta)\sin(\zeta) \end{bmatrix} \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix}. \quad (10)$$

Equation (10) provides the transformation from the aircraft's coordinate system to the radar's coordinate system which is the inverse of the desired transformation. Multiplying both sides by  $J^{-1}$  yields:

$$\begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix} = J^{-1} \begin{bmatrix} x \\ y \\ z \end{bmatrix}. \quad (11)$$

Normally, computing the inverse of a matrix with complicated terms like the ones in  $J$  would be quite involved; however, since this is a linear orthogonal transform, the inverse of the transformation matrix is equal to the transpose of the transformation matrix,  $J^{-1} = J^T$ . And correspondingly:

$$\begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix} = \begin{bmatrix} \cos(\xi')\cos(\eta) \\ \cos(\xi')\sin(\eta)\sin(\zeta) + \sin(\xi')\cos(\zeta) \\ -\cos(\xi')\sin(\eta)\cos(\zeta) + \sin(\xi')\sin(\zeta) \\ -\sin(\xi')\cos(\eta) \\ -\sin(\xi')\sin(\eta)\sin(\zeta) + \cos(\xi')\cos(\zeta) \\ \sin(\xi')\sin(\eta)\cos(\zeta) + \cos(\xi')\sin(\zeta) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}. \quad (12)$$

This completes all the pieces needed to transform  $\hat{V}_{RA}$  into  $\hat{V}'_{AR}$ . This transform can be written as:

$$\hat{V}'_{AR} = J^{-1}(-\hat{V}_{RA}) \quad (13)$$

Which, when substituting in Equation (4) and expanding, yields:

$$\begin{aligned} V'_{ARx} &= -\cos(\xi')\cos(\eta)\sin(Az)\cos(EI) + \sin(\xi')\cos(\eta)\cos(Az)\cos(EI) \\ &\quad -\sin(\eta)\sin(EI) \end{aligned} \quad (14a)$$

$$\begin{aligned} V'_{ARy} &= -(\cos(\xi')\sin(\eta)\sin(\zeta) + \sin(\xi')\cos(\zeta))\sin(Az)\cos(EI) \\ &\quad -(-\sin(\xi')\sin(\eta)\sin(\zeta) + \cos(\xi')\cos(\zeta))\cos(Az)\cos(EI) \\ &\quad +\cos(\eta)\sin(\zeta)\sin(EI) \end{aligned} \quad (14b)$$

$$\begin{aligned}
V'_{ARz} = & -(-\cos(\xi')\sin(\eta)\cos(\zeta) + \sin(\xi')\sin(\zeta))\sin(Az)\cos(EI) \\
& -(\sin(\xi')\sin(\eta)\cos(\zeta) + \cos(\xi')\sin(\zeta))\cos(Az)\cos(EI) \\
& -\cos(\eta)\cos(\zeta)\sin(EI).
\end{aligned} \tag{14c}$$

These equations can then be substituted into Equations (1) and (2) for the final aspect angles calculation.

#### IV. PRACTICAL CONSIDERATIONS FOR ASPECT ANGLES CALCULATION

In the above derivation, complete knowledge of the orientation vector of the aircraft was assumed. For a well instrumented test aircraft, this is a valid assumption; unfortunately, it breaks down in a combat situation. In this case, the aircraft's orientation vector must be calculated from the radar's positional measurements.

The need to calculate an aircraft's complete orientation vector from radar positional measurements presents some difficult problems in the roll component. Calculating the aircraft's roll angle would require a quadratic curve fitter, good estimates of the aircraft's velocity, and turn radius versus roll angle curves for all classes of aircraft. Coupling this computational complexity with the fact that most high roll maneuvers occur in an end game scenario when weapons have already been committed, leads to the conclusion that in an application sense it is useful and reasonable to set the roll component to zero.

Once this simplification has been made the heading and pitch components of the orientation vector can be calculated from simple linear travel. Let  $P_c(Rng, Az, EI)$  be the radar's current measurement of the aircraft's position w.r.t. radar, and let  $P_p(Rng, Az, EI)$  be the radar's previous measurement of the aircraft's position. Then, using Equation (3), the aircraft's motion can be described by:

$$\Delta x = x_c - x_p \tag{15a}$$

$$\Delta y = y_c - y_p \tag{15b}$$

$$\Delta z = z_c - z_p. \tag{15c}$$

From this, the pitch and heading can be calculated as:

$$\xi = \tan^{-1}(\Delta y / \Delta x) \tag{16}$$

$$\eta = \tan^{-1} \left( \frac{\Delta z}{\sqrt{(\Delta x)^2 + (\Delta y)^2}} \right). \tag{17}$$

It is worth noting that error components, unrelated to roll, do exist in these calculations. The velocity based heading angle will differ from the actual heading angle by an amount known as the crab angle. The crab angle is defined as the difference between an aircraft's direction of travel and the projection of its centerline and is typically caused by wind velocity. This angle is small at high aircraft velocities but can induce errors at lower velocities. In light of the large variation in wind velocities at various altitudes, this error cannot be readily predicted. Also, the actual pitch angle will vary from the calculated climb angle by an amount known as the Angle-of-Attack (AOA). The AOA results from the need for jet aircraft to pitch up in order to maintain flight. This angle varies with velocity and airframe but could be predicted once the data is provided to an NCTI algorithm and the airframe is estimated in the algorithm. In general though, the use of climb angle for pitch angle is sufficient.

The primary advantage of setting the roll component to zero is this allows a major simplification of the aspect angle problem. Under this condition, the problem becomes one of simple geometry and trigonometry as can be seen in Figures 5 and 6. Using these figures, the new equations for  $\theta$  and  $\phi$  are seen to be:

$$\theta = \beta - \xi \quad (18)$$

$$\phi = -1(\eta \cos(\theta) + El) \quad (19)$$

where:

$\beta = -1 * Az$ , which is the back azimuth from the aircraft to the radar,  
 $El$  is the elevation angle of the aircraft w.r.t. the radar.

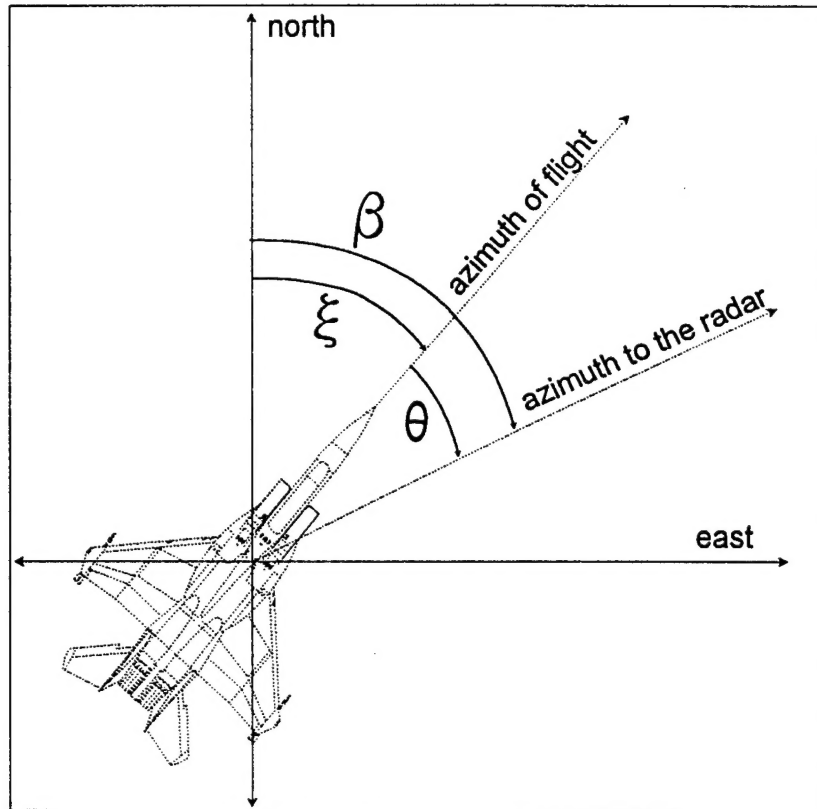


Figure 5. Simplified Estimate of  $\theta$

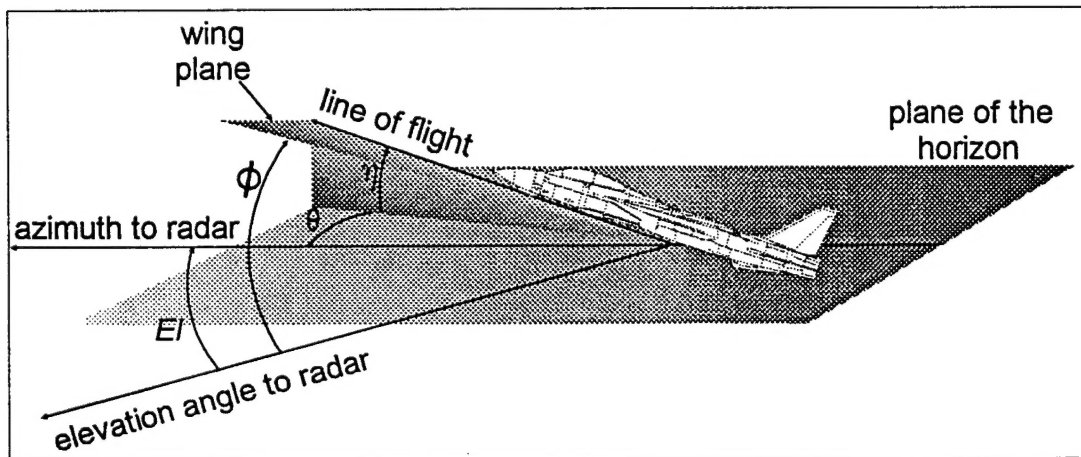


Figure 6. Simplified Estimate of  $\phi$

As will be shown in the next section, this method has been found to be within  $\pm 5$  degrees of the actual value for either aspect angle. Given that most NCTI algorithms search over at least this

much angle space, this is not an unreasonable result. This is especially true since it is not clear at this time if the roll component can be calculated with sufficient accuracy to improve these bounds.

## V. AIRCRAFT TESTS INVOLVING ASPECT ANGLES

NCTI research and development tests were performed using the Multi-Role Survivable Radar (MRSR) at Redstone Arsenal, AL. The aircraft involved were the F-15A and F-14A. MRSR is a highly accurate, 3-Dimensional (3-D) (provides Rng, Az, and El) radar which provides tracking and high range resolution data at a 2 second rate. As an input to NCTI algorithms, MRSR must provide an estimate of the target's aspect angles. In order to evaluate the accuracy of this estimate, positional and attitude truth data had to be collected.

The best available method for obtaining positional and attitude truth data is the Advanced Range Data System (ARDS). This is a precise (encrypted P-code) Global Positioning System (GPS) receiver integrated with an Inertial Reference Unit (IRU). The system collects positional and attitude data at a 10 Hz rate. When used with a reference receiver to provide differential corrections, this system can provide approximately 1 meter positional accuracy and approximately 1 degree angular accuracy in the roll, pitch, and yaw attitude angles. When operated in non-differential mode, the positional accuracy degrades to approximately 16 meters with no degradation in attitude accuracy.

Both aircraft in the test were outfitted with an ARDS unit contained in an AIM-9 pod and operated in the non-differential mode. MRSR track and ARDS data were collected on these aircraft. The aircraft were flown in racetrack patterns around the radar so that data over a wide range of aspect angles were collected. Given the accuracies of the ARDS unit, the aspect angles calculations should be accurate to approximately 1 degree in  $\theta$  and  $\phi$ . The error induced by the positional accuracy of non-differential, P-code GPS was insignificant for the target ranges.

The table shows the differences between the aspect angles as calculated from MRSR measured data using the equations in Section IV and as calculated from ARDS data using the equations in Section III. The aspect angles differences were also calculated with GPS positional data substituted for MRSR data in the equations in Section IV. This gives the best estimate of the errors resulting from the assumptions stated in Section IV for determining aspect angles with only positional information.

The table is also broken down into inbound and outbound data. This illustrates the effect of the aircraft's AOA above its line of flight. Note that the  $\phi$  error is positive (the underside of the aircraft is visible) on inbound runs and negative (the topside is visible) on outbound runs. This angle error is dependent on the type of aircraft, its velocity, and, obviously, on whether the aircraft is inbound or outbound. The velocity for these flights was kept at a constant 270 knots indicated air speed. An area for future research will be to attempt to develop a method to reduce this error given only the aircraft's velocity and length.



Table. Aspect Angle Accuracies

F-15 aspect angles measured by MRSR compared to IRU/GPS aspect angles			
$\theta_{in}$ avg err = $0.17^\circ$	$\phi_{in}$ avg err = $3.28^\circ$	$\theta_{out}$ avg err = $0.52^\circ$	$\phi_{out}$ avg err = $-2.41^\circ$
$\theta_{in}$ stdev = $4.52^\circ$	$\phi_{in}$ stdev = $5.14^\circ$	$\theta_{out}$ stdev = $3.86^\circ$	$\phi_{out}$ stdev = $4.83^\circ$
F-15 aspect angles measured by P-code GPS positional data compared to IRU/GPS aspect angles			
$\theta_{in}$ avg err = $-0.45^\circ$	$\phi_{in}$ avg err = $3.26^\circ$	$\theta_{out}$ avg err = $-.27^\circ$	$\phi_{out}$ avg err = $-2.89^\circ$
$\theta_{in}$ stdev = $2.20^\circ$	$\phi_{in}$ stdev = $2.18^\circ$	$\theta_{out}$ stdev = $2.73^\circ$	$\phi_{out}$ stdev = $1.68^\circ$
F-14 aspect angles measured by MRSR compared to IRU/GPS aspect angles			
$\theta_{in}$ avg err = $-0.45^\circ$	$\phi_{in}$ avg err = $1.97^\circ$	$\theta_{out}$ avg err = $-0.56^\circ$	$\phi_{out}$ avg err = $-4.82^\circ$
$\theta_{in}$ stdev = $4.21^\circ$	$\phi_{in}$ stdev = $5.46^\circ$	$\theta_{out}$ stdev = $4.36^\circ$	$\phi_{out}$ stdev = $5.79^\circ$
F-14 aspect angles measured by P-code GPS positional data compared to IRU/GPS aspect angles			
$\theta_{in}$ avg err = $-0.27^\circ$	$\phi_{in}$ avg err = $1.71^\circ$	$\theta_{out}$ avg err = $-0.27^\circ$	$\phi_{out}$ avg err = $-3.87^\circ$
$\theta_{in}$ stdev = $3.13^\circ$	$\phi_{in}$ stdev = $1.50^\circ$	$\theta_{out}$ stdev = $4.60^\circ$	$\phi_{out}$ stdev = $5.57^\circ$

# INITIAL DISTRIBUTION LIST

	<u>Copies</u>
IIT Research Institute ATTN: GACIAC 10 W. 35th Street Chicago, IL 60616	1
AMSMI-RD	1
AMSMI-RD-CS-R	5
AMSMI-RD-CS-T	1
AMSMI-RD-MG, Mr. Tracy Jackson	1
Mr. Rex Powell	1
AMSMI-RD-MG-RF, Mr. David Abell	3
Mr. Willie Caraway III	3
Dr. George Emmons	1
AMSMI-RD-MG-ST, Dr. Jay Loomis	1
AMSMI-GC-IP, Mr. Fred Bush	1